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LETTERS TO THE EDITOR

COMMENTS ON COUPLED FLEXURAL–TORSIONAL VIBRATIONS OF TIMOSHENKO BEAMS

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1. INTRODUCTION

In their paper, Bercin and Tanaka [1] studied mainly the warping effect in coupled bending–torsional vibration analysis of a Timoshenko beam. Also they have reviewed briefly a few groups of theories related with the subject. They stated correctly that unacceptably large errors in the determination of natural frequencies could arise when the bending shear effect and the warping flexibility are neglected. Although many authors have focused their attention on the important role of the warping effect both in static and dynamic problems [2–5], it has to be noted that few of them have taken into account the shear flexibility due to warping, which could have a noticeable influence in many cases.

Cortínez and Rossi [6] developed a dynamical theory of thin walled open section straight beams, which considers the shear effect mentioned above. Also Wu and Sun [7] have developed a theory including the shear flexibility due to warping. Some other extensions were made for curved beams [8–10], including other second order effects such as curvature.

In the case of a monosymmetric thin-walled beam, the motion equations governing the coupled bending–torsional vibration including shear flexibility due to warping [6], could be reduced to

$$-GK_y(v'' - \theta_z') + \rho A(\ddot{v} - z_0\ddot{\phi}) = 0, \quad - EI_z\theta_z'' - GK_y(v' - \theta_z) + \rho I_z\ddot{\theta}_z = 0, \quad (1, 2)$$

$$-EC_w\theta'' - GK_w(\phi' - \theta) + \rho C_w\ddot{\theta} = 0, \quad (3)$$

$$-GK_w(\phi'' - \theta') - GJ\phi'' + \rho(I_s\ddot{\phi} - Az_0\ddot{v}) = 0, \quad (4)$$

where primes and dots denote spatial and temporal derivatives respectively, and $v(x, t)$ is the transverse displacement perpendicular to the symmetry axis, $\theta_2(x, t)$ is the bending slope, $\phi(x, t)$ is the torsional rotation, $\theta(x, t)$ is a function that

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TABLE 1

Comparison of natural frequencies (Hz) *of a cantilever U-beam*; (I) *reference* [1], (*Table* 3, *case* 2); (II) *present model allowing warping shear flexibility*

	Approach	
Modal number	(I)	(II)
1	23·79	23·21
2	78·26	75·20
3	124·78	109·92
4	295·26	251·12
5	334·88	290·35

describes the warping intensity along the beam, EI_z, EC_w and GJ are respectively, the bending, warping and torsional rigidities, whereas GK_y and GK_w are the bending and warping shear rigidities, ρ is the density, I_z is the moment of inertia of the cross-section with respect to the z-axis, I_s is the polar moment of inertia with respect to the shear center, C_w is the warping constant and A the sectional area.

The difference between the present equations with those of reference [1] lies in the torsion motion equations, which are split into warping motion and torsion motion equations. The shear flexibility due to warping [6] affects the coupling conditions in the differential equations system, and it has important consequences when high modes of vibration have to be calculated and especially when short beams are involved. To show this effect, coupled bending–torsional frequencies of a few sections are given for clamped–free boundary conditions by including or excluding the warping shear effect as well as the associated warping inertia. Exclusion of rotary and warping inertias and shear effects reduce the model (1–4) to the Vlasov beam theory [6].

TABLE 2

Comparison of natural frequencies (Hz) *of a cantilever semi-circular beam*; (1) *reference* [1] (*Table* 2, *case* 3); (2) *present model allowing warping shear flexibility*, (3) *Vlasov model*

		Modal number				
h/L	Case	1	2	3	4	5
0·059	(1)	63·51	137·39	275·82	481·10	639·76
	(2)	63·37	137·24	274·10	473·53	640·06
	(3)	63·69	137·79	277·07	479·19	664·49
0·120	(2)	180·99	403·73	648·31	1203·31	1922·73
	(3)	182·76	412·37	661·72	1239·77	2034·36

TABLE 3

Comparison of natural frequencies (Hz) *of a cantilever box-beam*; (1) *reference* [1] (*Table* 4, *case* 2); (2) *present model allowing warping shear flexibility*; (3) *Vlasov model*

		Modal number				
h/L	Case	1	2	3	4	5
0·034	(1)	11·01	38·93	57·82	150·51	205·32
	(2)	10·97	38·15	56·62	143·14	210·46
	(3)	11·03	39·42	58·29	152·26	211·53
0·100	(2)	71·66	259·03	362·56	842·81	1237·00
	(3)	74·24	274·18	442·17	1226·99	1674·03

2. NUMERICAL COMPARISONS

Numerical determinations of natural frequencies are performed by means of the finite element method [6], by employing 20 elements of equal length. Table 1 depicts the natural frequencies corresponding to the Example II of reference [1] compared with frequencies obtained by a finite element approach of the model (equations 1–4) including shear flexibility due to warping. Table 2 shows the first five natural frequencies of a cantilever beam with the semi-circular cross-section of Example I of reference [1]. In this table, the case of a beam shorter ($h/L = 0{\cdot}120$) than that used in reference [1] (actually $h/L = 0{\cdot}059$) is included. Table 3 shows basically the same information of Table 2, but for the box section with slit, corresponding to the Example III of reference [1]. In Table 4 the first five torsional frequencies for an I-beam are presented, including and excluding shear flexibility due to warping. In this last example, the sectional properties considered were: flange width 0·6 m, web height 0·6 m, thickness 0·03 m, length 4·0 m, modulus of elasticity $2{\cdot}1 \times 10^{11}$ N/m^2, Poisson coefficient 0·3 and density 7830 kg/m^3.

TABLE 4

Comparison of natural torsion frequencies (Hz) *of a cantilever I-beam*; (I) *present model excluding warping shear flexibility*; (II) *present model allowing warping shear flexibility*

	Approach	
Modal number	(I)	(II)
1	30·00	29·50
2	165·80	150·50
3	453·90	370·60
4	884·80	642·30
5	1459·90	945·50

3. CONCLUSIONS

From the numerical comparisons performed it could be concluded that the shear flexibility due to warping has a noticeable influence in higher modes, or even in lower modes in the case of deep beams. Its main effect is to decrease the value of the corresponding frequency. The warping shear flexibility is strongly dependent on the section shape and the height–length ratio. Particularly, in the case of a semi-circular section, there is no great difference between including or excluding shear flexibility due to warping. The explanation of this behavior may be found in the fact that the Saint Venant torsion mechanism is dominant (compared with the warping torsion) in this case. However the circumstances are different for the other cases, as was shown.

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